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CONSIGLIO NAZIONALE DELLE RICERCHE
Istituto di Fotonica e Nanotecnologie

Talbot effect in X-Ray Waveguides

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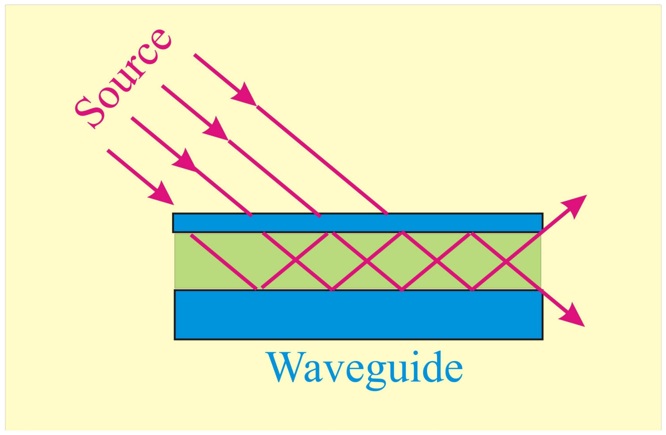
SHORT
WAVELENGTH
LABORATORY
SOURCES
COST MP0601

Poland,Krakow
2010

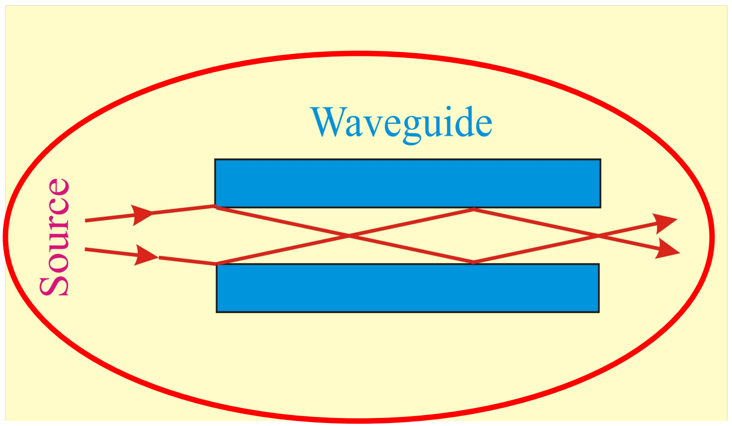
COST Action MP0601

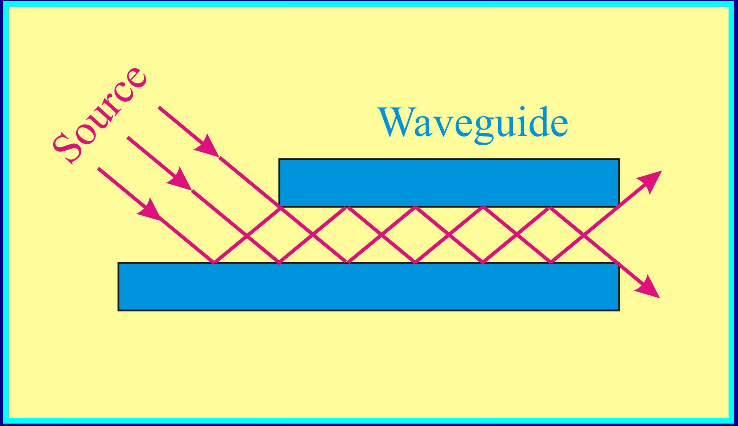
Waveguides

Resonant beam coupling



Front-coupling





Advantages of front-coupling WG

- Core layer: vacuum
- FC WG as optics for X-ray tubes

Computer simulation

Source

- incoherent radiation
- coherent radiation

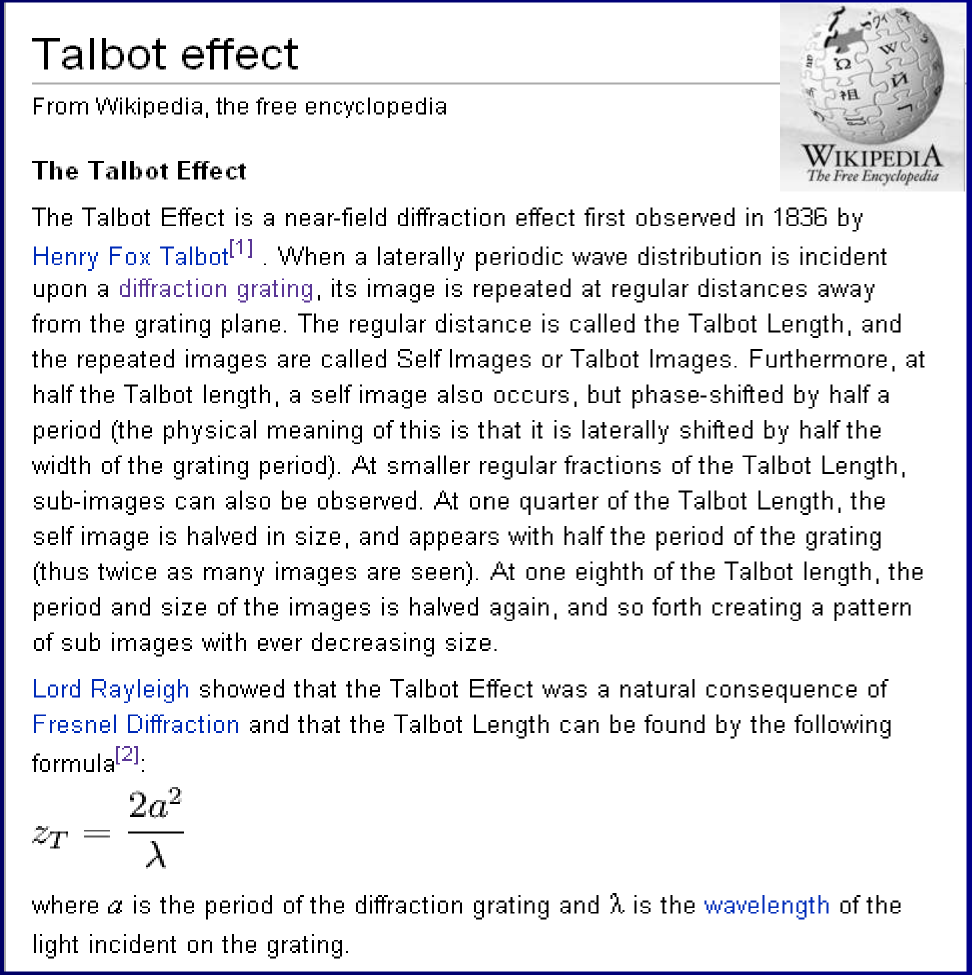
Optics

- Waveguide with front-coupling

Method

- **Numerical solution of wave equation**
- **Analytical solution of wave equation**

Talbot effect in periodical structures



Talbot effect

From Wikipedia, the free encyclopedia

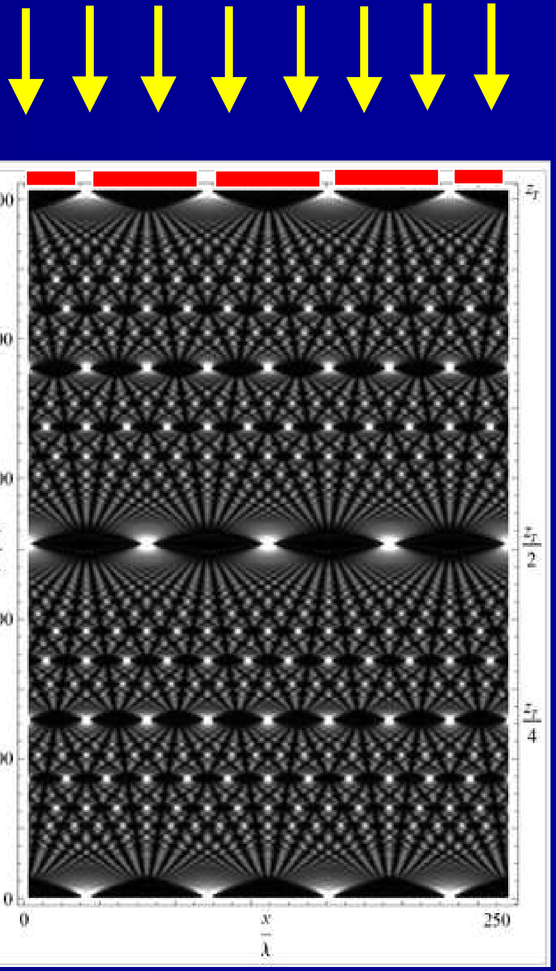
The Talbot Effect

The Talbot Effect is a near-field diffraction effect first observed in 1836 by Henry Fox Talbot[1] . When a laterally periodic wave distribution is incident upon a diffraction grating, its image is repeated at regular distances away from the grating plane. The regular distance is called the Talbot Length, and the repeated images are called Self Images or Talbot Images. Furthermore, at half the Talbot length, a self image also occurs, but phase-shifted by half a period (the physical meaning of this is that it is laterally shifted by half the width of the grating period). At smaller regular fractions of the Talbot Length, sub-images can also be observed. At one quarter of the Talbot Length, the self image is halved in size, and appears with half the period of the grating (thus twice as many images are seen). At one eighth of the Talbot length, the period and size of the images is halved again, and so forth creating a pattern of sub images with ever decreasing size.

Lord Rayleigh showed that the Talbot Effect was a natural consequence of Fresnel Diffraction and that the Talbot Length can be found by the following formula[2]:

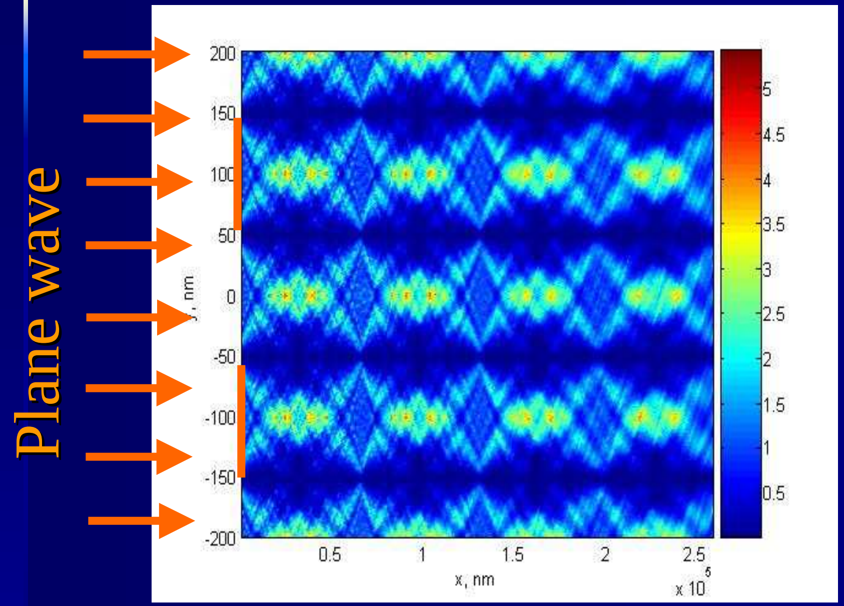
$$z_T = \frac{2a^2}{\lambda}$$

where a is the period of the diffraction grating and λ is the wavelength of the light incident on the grating.

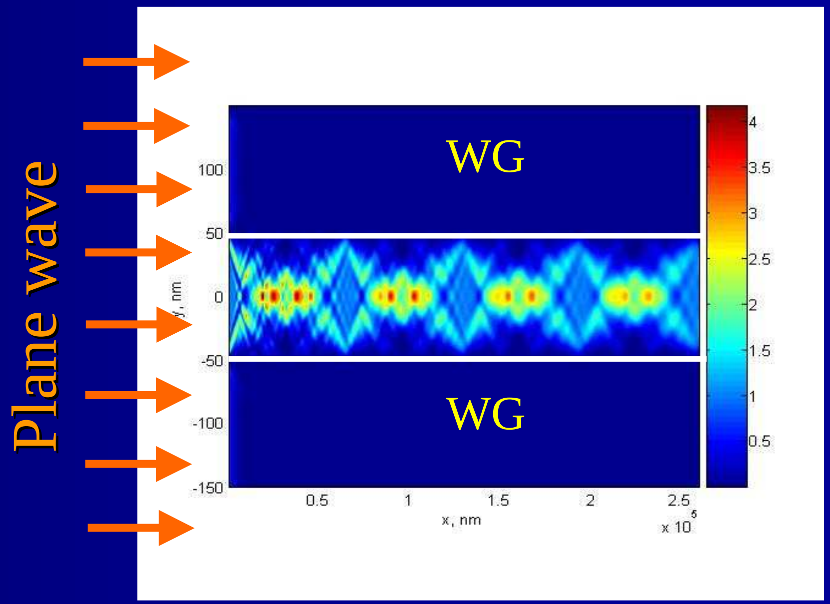


Talbot effect. Phase gratings (phase π) vs plane FC WGs

In general self image fenomenon occur in wave field composed of discrete modes



Phase grating (π)
Period D=2d=200 nm



Waveguide with vacuum gap
$d=100$ nm, $\lambda=0.154$ nm, $R_{fr}=1$

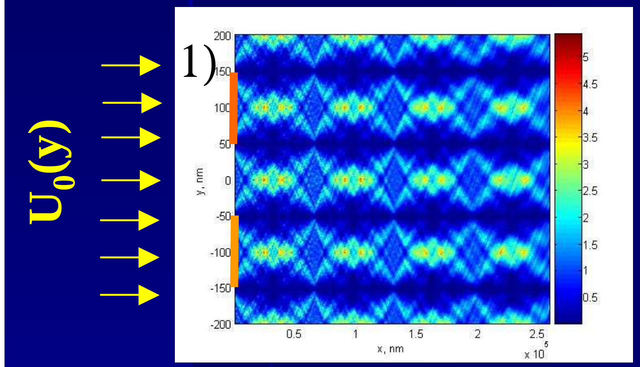
The interference pattern has a maximum modulation at distances

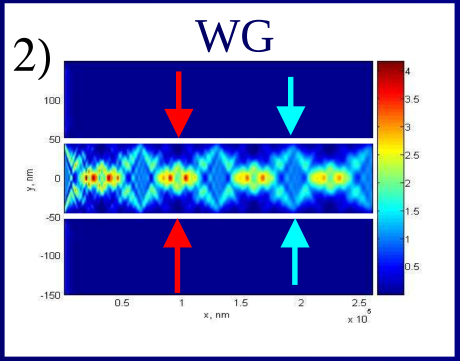
$X_m=mD^2/(8\lambda)$ – fractional Talbot distance, for WG $D=2d$

Talbot effect. Phase gratings (phase π) vs plane FC WGs

Diffraction pattern has a maximum modulation at fractional Talbot distances $X_T = mD^2/8\lambda$

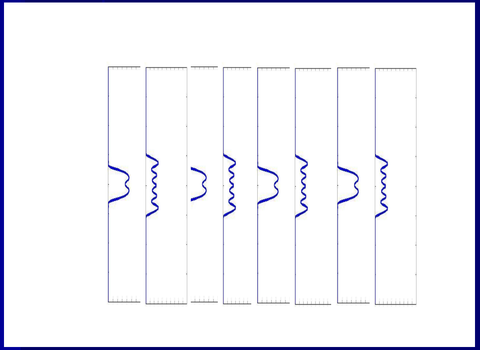
Au, $\lambda = 0.154$ nm d = 100 nm

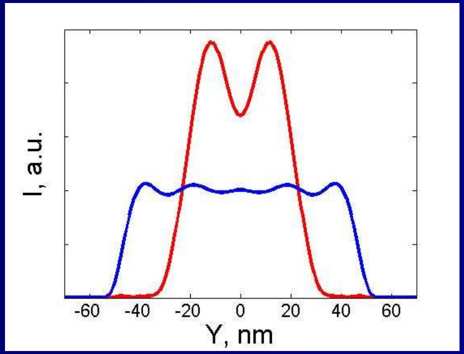




1) **Phase grating. Period D, 0.5 duty cycle and π phase shift. Diffraction pattern has half the period of the grating d=D/2**

2) **FC waveguide with vacuum gap d=D/2.**

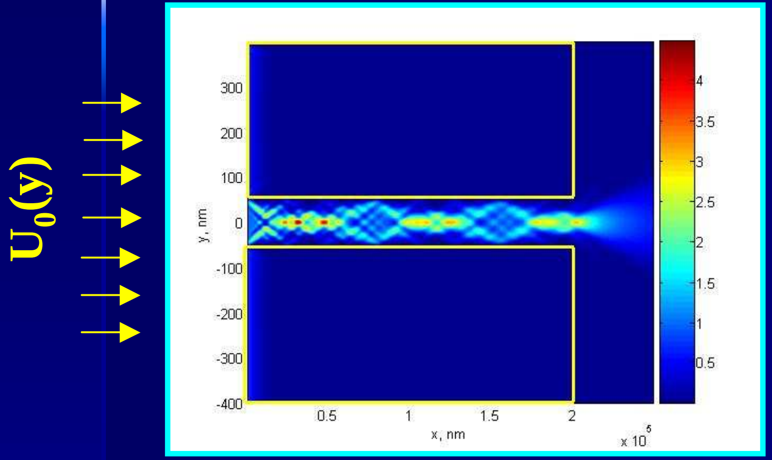


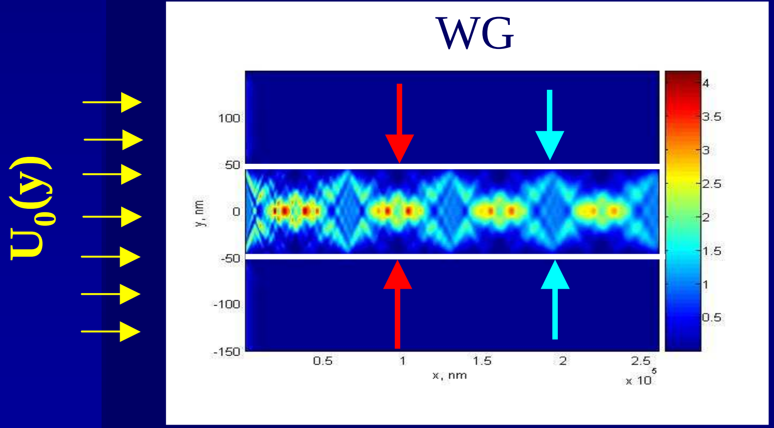


The width of the energy distribution at odd fractional Talbot distances $X_T = mD_{eff}^2/8\lambda$ is one half that at even distances

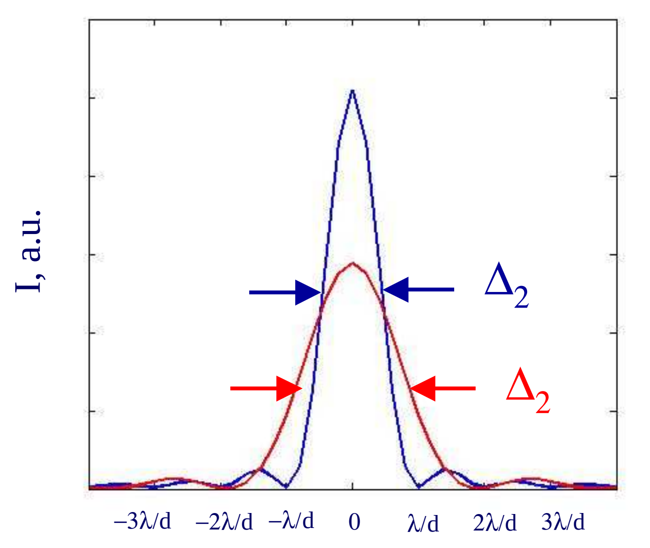
$d_{eff} \approx d + 2\zeta$, where $\zeta = 1/k(\theta c - \theta_m)^{1/2}$ is the penetration depth for m-th mode

Talbot effect.
Front coupling WGs





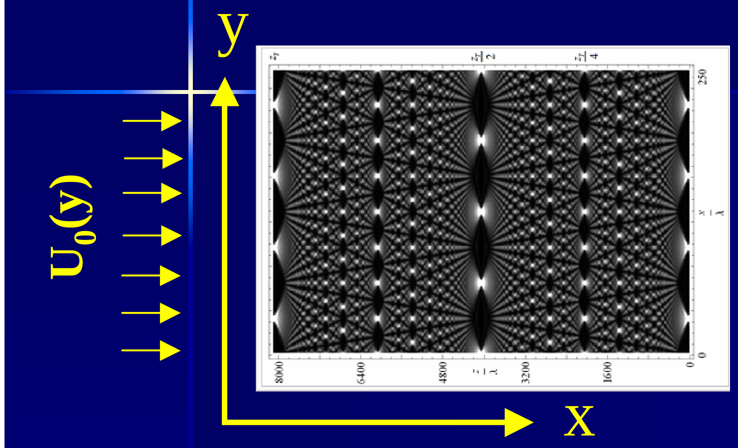
Diffraction pattern in the far field zone



The width of the energy distribution in far field zone for even fractional Talbot distances is one half that for odd distances

$\Delta_2 \sim 2\,\Delta_1$

Montgomery self-imaging



Wave field with a lateral periodicity P_y is periodic in longitudinal direction with period

Talbot self-imaging (sufficient condition)

$$U_0(y) = U_0(y + P_y) \Rightarrow U_x(y) = U_{x+P_x}(y)$$

$$P_x = \frac{2P_y^2}{\lambda}$$

the paraxial case

Montgomery self-imaging (necessary condition)

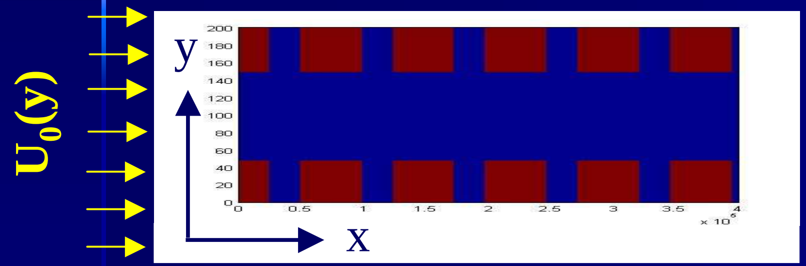
Wave field has a longitudinal periodicity Px in direction x if the lateral components of the k vector obeys the condition

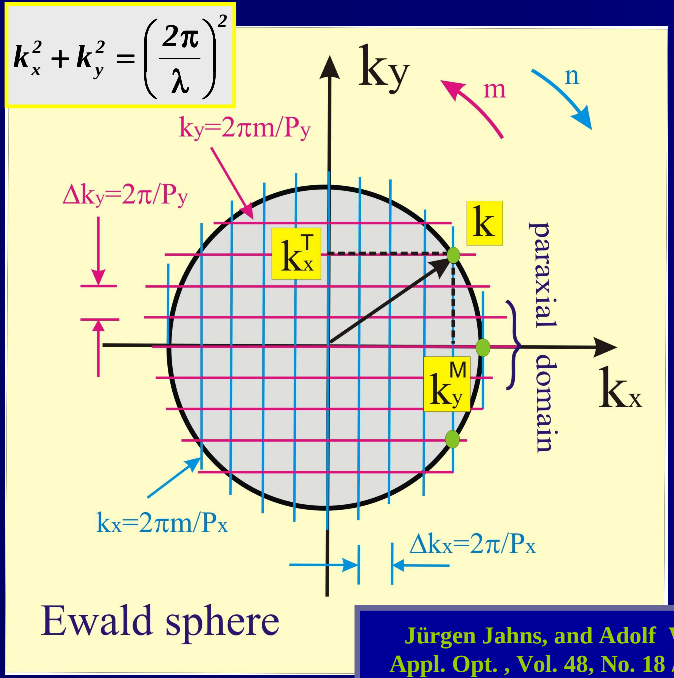
$$U_x(y) = U_{x+Px}(y) \Rightarrow U_0(y) = \sum_m A_m \exp(ik_{y,m}y)$$

$$k_{y,m}^2 = (2\pi)^2\left[\left(\frac{1}{\lambda}\right)^2 - \left(\frac{m}{P_x}\right)^2\right]$$

Montgomery self-imaging

WG with the longitudinal periodicity





$$k_x^2 + k_y^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

1. Wave field with lateral periodicity

$$k_{T,x}^m = \pm 2\pi\sqrt{\left(\frac{1}{\lambda}\right)^2 - \left(\frac{m}{P_y}\right)} \approx 2\pi\left(\frac{1}{\lambda} - \frac{m^2}{P_y}\right), \quad m = 0, \pm 1, \pm 2, \ldots$$

2. The longitudinally periodic wave field

$$k_{M,y}^n = \pm 2\pi\sqrt{\left(\frac{1}{\lambda}\right)^2 - \left(\frac{n}{P_x}\right)^2}, \quad n = \pm 1, \pm 2, \ldots$$

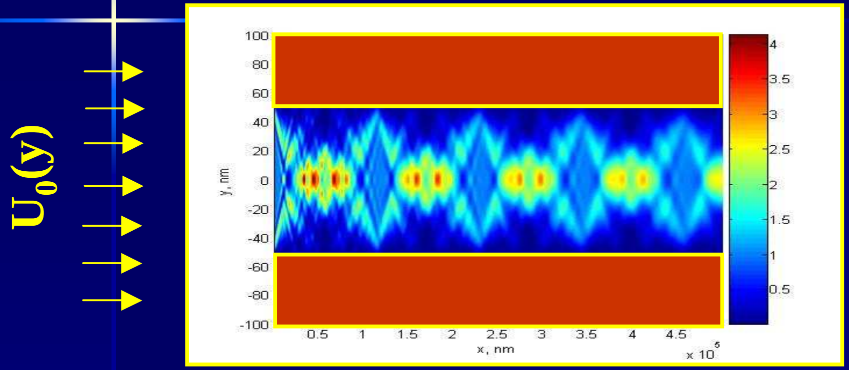
3. Lateral P_y and longitudinal period P_x are varied independently

$$\left(k_y^M\right)^2 + \left(k_x^T\right)^2 = \left(\frac{2\pi}{\lambda}\right)^2 \longrightarrow m^2\left(\frac{\lambda}{2d}\right)^2 + n^2\left(\frac{\lambda}{P_x}\right)^2 = 1$$

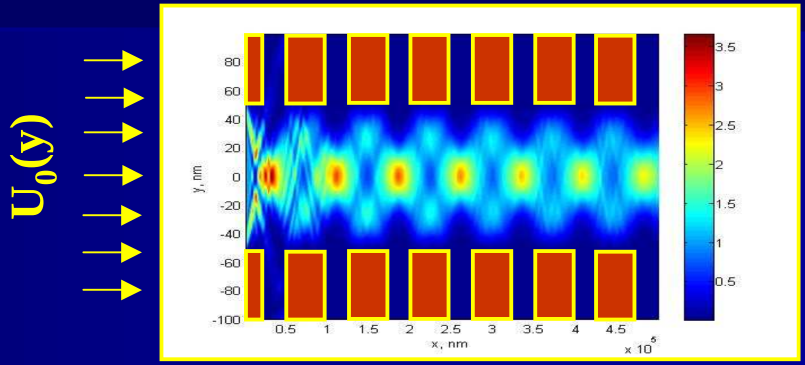
Jürgen Jahns, and Adolf W. Lohmann
Appl. Opt. , Vol. 48, No. 18 / 20 June 2009

Paraxial domain $m < m_{\max}, \quad n \sim N \approx p_x / \lambda$

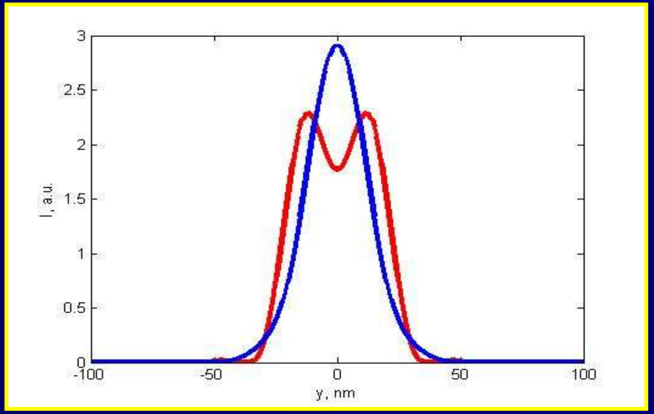
Multimodal WGs with longitudinal periodicity



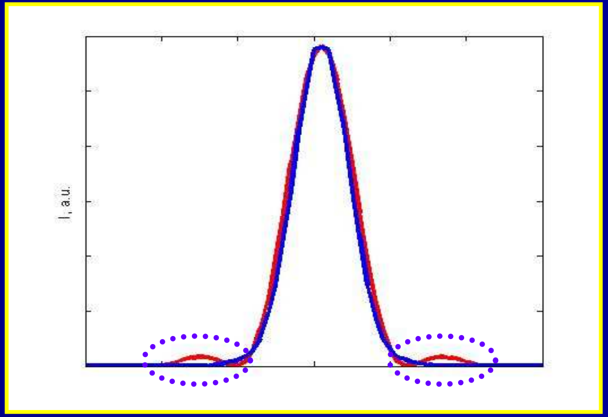
Multimodal WG, Au, λ=0.1nm,d=100 nm



WG with longitudinal periodicity.
Period $P_L=d_{eff}^2/\lambda$, duty 2/3

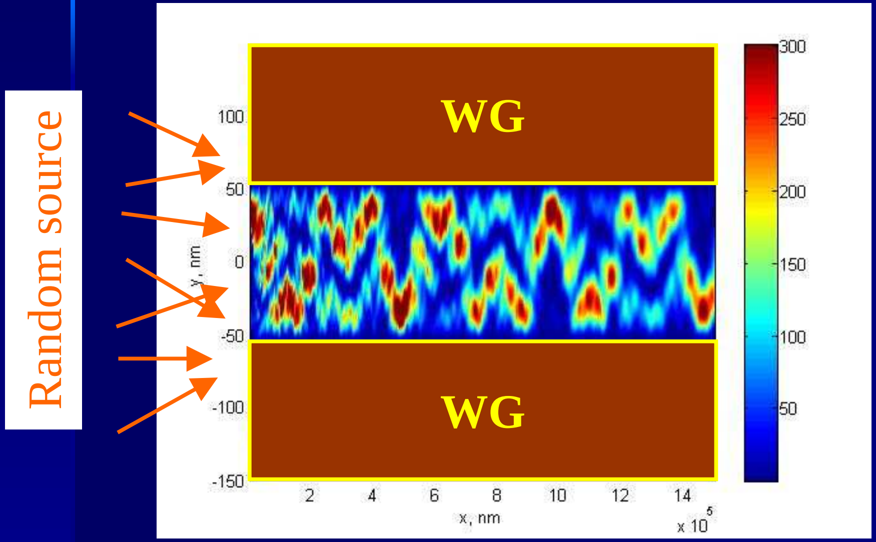


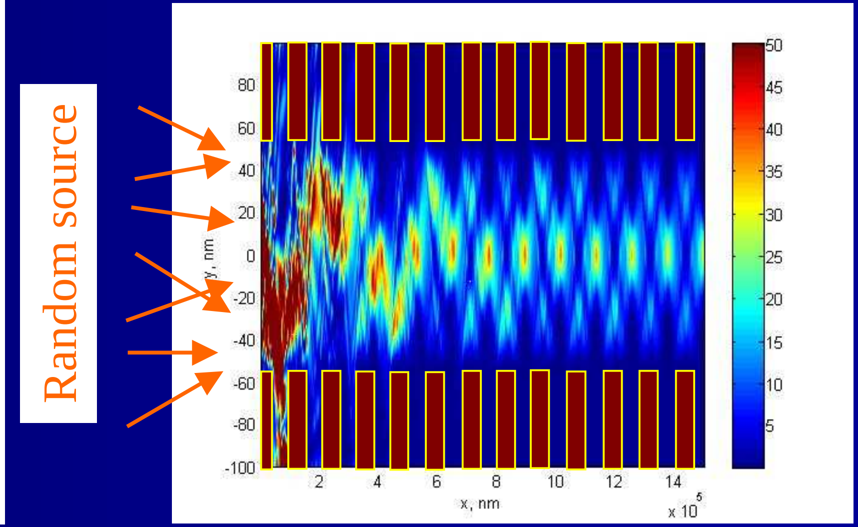
The energy distribution at exit apertue of the WG (red line) and WG with grating (blue line)



The energy distribution at far field zone. WG (red line) and WG with grating (blue line)

Multimodal WGs with longitudinal periodicity. Incoherent source.





Propagation of the incoherent wave in the WG

WG with grating.
Period $P=deff^{2}/\lambda=119.9\ \mu m$
(39.97 µm vacuum, 79.94 µm Cr)
d_{eff} =109.5 nm

Conclusion

1. The field in multimode x-ray waveguides with longitudinal periodicity are studied.

2. Modal structure of WGs depends on the ratio of the lateral and longitudinal periods

3. Montgomery condition for the wave vector includes the wavelength of the x-ray radiation and therefore x-ray waveguides with longitudinal periodicity can influence on temporal frequencies of the field

Thank you